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APPLICATION OF PHOTOELASTIC COATINGS IN THE INVESTIGATION OF SHELLS

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APPLICATION OF PHOTOELASTIC COATINGS IN THE INVESTIGATION OF SHELLS

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ABSTRACT

The article examines stress and resultant membrane and flexural deformation in coatings. Thickness and rotation direction are considered.

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It is shown that errors introduced by the rotation of the principal directions of deformation along the thickness of the coating depend on the relationship between the thickness of the shells and of the coatings and that they may be reduced to permissible values by decreasing the thickness of the coatings. /84*

The state of stress of a photoelastic coating applied to a thin shell may be represented as the sum of the membrane and flexure states of deformation. In this case, if the principal deformations of both states do not coincide in direction the principal directions of the resulting state of strain will vary along the thickness of the coating. Reference 1 has noted several cases when there is a substantial rotation of the direction of

*Numbers given in margin indicate pagination in original foreign text.

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principal deformations along the thickness of the coating and has concluded that substantial errors may arise if this rotation is neglected. By using the theory of characteristic directions developed by the author (ref. 1) and assuming a uniform rotation of the principal directions and also assuming a constant difference between the principal deformations along the thickness of the coating, it is shown that a decrease in the thickness of the coating produces practically no decrease in the error introduced by the rotation of the principal directions; this conclusion requires a correction.

1. The directions of the principal deformations in the coating, measured from the direction of the principal membrane deformation with high algebraic value, are given by the expression (refs. 1 and 2)

$$\Phi = 0.5 \operatorname{Arc\,tg} \frac{kz \sin 2\beta}{t + kz \cos 2\beta} \qquad \left(k = \frac{(\varepsilon_1 - \varepsilon_2)_m}{(\varepsilon_1 - \varepsilon_2)_n}\right) \tag{1.1}$$

Here $(\epsilon_1 - \epsilon_2)_m$ is the difference between the principal flexural deformations on the surface of the shell, $(\epsilon_1 - \epsilon_2)_n$ is the difference between the principal membrane deformations, β is the angle between the high algebraic values of the principal membrane and flexural deformations; the other designations are the same as those given in reference 1. The angle between the directions of the principal deformations on the external and internal surfaces of the coating are determined from (1.1)

$$\Phi_0 = \frac{d}{2t} \frac{k \sin 2\beta}{1 + 2k\eta_0 \cos 2\beta + k^2\eta_0^2} \qquad \left(\eta_0 = \frac{z_0}{t}\right) \tag{1.2}$$

Here d is the thic[illegible]elastic coating. We shall follow reference 1 and assume that rotation is uniform. We may assume that

$z_0 = t + 0.5\ d$ is the coordinate of the middle surface of the coating. The difference between the principal deformations in the coating (ref. 2) is given by

$$\varepsilon_1 - \varepsilon_2 = (\varepsilon_1 - \varepsilon_2)_n\,(1 + 2k\eta\cos 2\beta + k^2\eta^2)^{1/2} \tag{1.3}$$

From (1.2) we find that the maximum rotation will take place when $k = \pm t/z_0$. Then

$$\Phi_{0\,max} = \frac{d}{4t}\operatorname{tg}\beta \quad (z > 0), \qquad \Phi_{0\,max} = \frac{d}{4t}\operatorname{ctg}\beta \quad (z < 0) \tag{1.4}$$

Since it has been stipulated that $45^\circ \leq \beta \leq 45^\circ$ (ref. 1), a slight rotation of the principal deformations in the coating applied to the surface $z > 0$ (where the membrane and the flexural deformations are added in absolute value) cannot exceed the quantity $0.5\ N^{-1}$ (N is the ratio of the shell thickness to the thickness of the coating). On the surface $z < 0$, the magnitude of rotation for small angles β may be substantial; however, this does not introduce substantial errors in determining the membrane or flexural deformations when using the method described in reference 3. This is so because when there are substantial rotations, the difference between the principal deformations in the coating $z < 0$ is rather small compared with the corresponding quantity in the coating $z > 0$. Indeed we can see from expression (1.3) that if $k = t/z_0$, $\beta \to 0$ then $\epsilon_1 - \epsilon_2 \to 0$ on the surface $z < 0$ and in the limit $\epsilon_1 - \epsilon_2$, i.e., any direction is the principal direction so that there is no error in its determination.

In this connection it is logical to evaluate the discrepancy not from the magnitude of the error in determining the directions of principal

deformations but rather from the magnitude of the error in determining the component of the membrane and flexural states of strain.

2. In determining these errors we shall assume that the characteristic directions coincide with the directions of the principal stress axes (strain axes), by rotating together with them (ref. 1). This is also confirmed by experiments conducted to investigate the isoclines in the field of a crossed polariscope with a source of white light when using epoxy type coatings (ref. 3).

Then the relative error in determining the difference between the longitudinal membrane deformations, due to the incorrect determination of the directions of principal deformations, for the case when coatings of equal thickness are applied on both surfaces of the shell will be given by the /85 expression

$$\Delta(\varepsilon_x - \varepsilon_y)_n = 1 - \frac{(\varepsilon_1 - \varepsilon_2)^{(+)} \cos 2\Phi_*^{(+)} + (\varepsilon_1 - \varepsilon_2)^{(-)} \cos 2\Phi_*^{(-)}}{(\varepsilon_1 - \varepsilon_2)^{(+)} \cos 2\Phi^{(+)} + (\varepsilon_1 - \varepsilon_2)^{(-)} \cos 2\Phi^{(-)}} \quad (2.1)$$

Here $(\varepsilon_1 - \varepsilon_2)^{(+)}$ and $(\varepsilon_1 - \varepsilon_2)^{(-)}$ are the differences between the principal deformations in the middle surfaces of coatings $z > 0$ and $z < 0$, $\Phi^{(+)}$ and $\Phi^{(-)}$ are the directions of these deformations, $\Phi^{(+)}$ and $\Phi^{(-)}$ are the directions of the principal deformations on the external surface of the coatings $z > 0$ and $z < 0$. Utilizing (1.1) and (1.3) we obtain

$$\Delta(\varepsilon_x - \varepsilon_y)_n = 1 - {}^1/_2\,[(1 + k\cos 2\beta)\,\omega^{(+)} + (1 - k\cos 2\beta)\,\omega^{(-)}] \quad (2.2)$$

In a similar manner

$$\Delta(\varepsilon_x - \varepsilon_y)_m = 1 - \frac{t}{2kz_0 \cos 2\beta}\,[(1 + k\cos 2\beta)\,\omega^{(+)} - (1 - k\cos 2\beta)\,\omega^{(-)}] \quad (2.3)$$

$$\Delta\gamma_{xy,m} = 1 - \frac{t}{2z_0}\,[\omega^{(+)} + \omega^{(-)}], \quad \Delta\gamma_{xy,n} = \frac{1}{2}k\sin 2\beta\,[\omega^{(+)} - \omega^{(-)}] \quad (2.4)$$

Here $\omega^{(+)}$ and $\omega^{(-)}$ is the ratio of the differences of the principal deformations in the middle surfaces to their differences on the external surfaces of the coatings $z > 0$ and $z < 0$ respectively, $\Delta\,(\epsilon_x - \epsilon_y)_m$ and $\Delta\gamma_{xy,m}$ are the relative errors in determining the differences of flexural longitudinal and sheer deformations. The error in the sheer membrane deformations $\Delta\gamma_{xy,n}$ is referred to the difference of the corresponding longitudinal deformations (the axes x and y coincide with the principal axes of the membrane deformations).

Expressions (2.2) - (2.4) show that as the thickness of the coatings decreases, the error introduced by the rotation of the principal deformations in the coating decreases and is absent in the limit since $\omega^{(+)} \to 1$, $\omega^{(-)} \to 1$, $t/z_0 \to 1$.

We note that if by following ref. 1 we neglect the nonuniformity in the distribution of deformations along the thickness of the coating ($\omega^{(+)} = \omega^{(-)} = 1$) or if we assume that rotation is strictly uniform ($d\Phi/dz = \text{const}$), then in general the error introduced by the incorrect determination of the directions of principal deformations will be absent if the data which are obtained are referred to the external surface of the coating.

For the case of maximum rotations $k = t/z_0$ we may obtain approximate expressions for determining the errors if in deriving them we neglect the term N^{-2} compared with unity

$$\Delta\,(\varepsilon_x - \varepsilon_y)_n = \Delta\gamma_{xy,m} = 1 - (1 - N^{-1})^{1/2}, \qquad \Delta\,(\varepsilon_x - \varepsilon_y)_m = 1 - (1 - N^{-1})^{-1/2} \tag{2.5}$$

Expressions (2.5) show that for $N \geq 10$ recommended in reference 3 the error does not exceed 6%. The values of the errors in percent incurred when determining the differences of membrane and flexural deformations for $N = 10$ are presented in the table for certain values of k and β. An analysis of this

β	$k = 0.5$		$k = 1.0$		$k = 2.0$	
	$\Delta(\varepsilon_x - \varepsilon_y)_n$	$\Delta(\varepsilon_x - \varepsilon_y)_m$	$\Delta(\varepsilon_x - \varepsilon_y)_n$	$\Delta(\varepsilon_x - \varepsilon_y)_m$	$\Delta(\varepsilon_x - \varepsilon_y)_n$	$\Delta(\varepsilon_x - \varepsilon_y)_m$
0	0	0	0	0	0	0
$^1/_{18}\pi$	0.5	1.0	5.0	3.0	1.5	1.0
$^1/_8\pi$	1.5	2.0	5.5	6.0	5.0	2.0
$^7/_{36}\pi$	2.0	2.0	6.0	6.0	7.0	3.0
$^1/_4\pi$	2.0	—	6.5	—	7.5	—

table shows that the maximum errors occur for small angles β, when there are substantial rotations in the layer $z < 0$ and for angles β close to 45° when, as we can see from expression (1.4), the angle between the directions of the principal deformations on the external and middle coating surfaces does not exceed 1/40 radians and is within the limits of accuracy of the directions of principal deformations when the coatings are investigated by the photoelastic method.

Thus when there are relatively large differences in the process, the error introduced by the discrepancy between the experimentally determined characteristic directions and the directions of the principal deformations in the middle surface of the coating, depends on the ratio of shell thicknesses to the coating thicknesses, and, in particular when $N = 10$, they are within the limits of experimental accuracy.

It is easy to extend this conclusion to the case of operation with small differences when the characteristic directions do not coincide with the directions of principal deformations on the external surface of the coating.

The experimentally determined characteristic directions measured /86 from the direction of the principal stresses at a point where light is introduced to the coating, for the case of the uniform rotation of the directions of principal stresses when their difference is constant, is given by expression (1)

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$$\mathrm{tg}\,2\alpha = \frac{2\Phi_0}{(\delta^2 + 4\Phi_0^2)^{1/2}}\,\mathrm{tg}\,\frac{(\delta^2 + 4\Phi_0^2)^{1/2}}{2}, \qquad \delta = \frac{2\pi}{\lambda} c\,(\varepsilon_1 - \varepsilon_2)\,d \tag{2.6}$$

As the thickness of the coating decreases there is a proportional decrease in the quantities Φ_0 and d (see (1.2) and (2.6)). For small thickness we can show that

$$\mathrm{tg}\,{}^1/_2\,(\delta^2 + 4\Phi_0^2)^{1/2} \approx {}^1/_2\,(\delta^2 + 4\Phi_0^2)^{1/2}, \text{ ст } \mathrm{tg}\,2\alpha \approx \Phi_0. \tag{2.7}$$

Consequently the characteristic directions coincide approximately with the directions of the principal deformations (stresses) in the middle surface of the coating.

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REFERENCES

1. Aben, Kh. K., The Application of Photoelastic Coatings in the Investi of Shells (K primeneniyu fotouprugikh pokrytiy dlya issledovaniya obolochek). Izv. AN SSSR, Mekhanika i Mashinostroyeniye, No. 6, 1964.
2. Froeht, M., Photoelasticity (Fotouprugost'). Gostekhizedat, Vol. 2, 1950.
3. Akhmetzyanov, M. Kh., The Application of the Method of Photoelastic Coatings for Determining the Stresses and Deformations in Flexible Plates and Shells (Primeneniye metoda fotouprugikh pokrytiy dlya opredeleniya napryazheniy i deformatsiy v gibkikh plitakh i obolochkakh). Izv. AN SSSR, Mekhanika i Mashinostroyeniye, No. 1, 1964.